(1) Suppose $p$ is prime. Show that $\sqrt{p}$ is irrational.

(2) If $r \neq 1$, show that $1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$.

(3) If $2^n - 1$ is prime, show that $n$ is prime. Hint: If $n = ab$, consider $\frac{(2^a)^b - 1}{2^a - 1}$.

(4) Suppose $S$ is a non-empty subset of the positive integers. Show that $S$ has a smallest element. [Hint: Suppose not. Then construct a chain $x_1 > x_2 > x_3 > \ldots$ of elements of $S$.]